

# **Monitoring and Exploiting Student Evaluations of University Courses and Faculty Members using Statistical Process Control Tools**

## **Abstract**

Student evaluations of faculty members' teaching effectiveness have recently become an issue of great importance in Higher Education (HE). In this paper, we develop a statistical framework, based mainly on Statistical Process Control techniques, which will enable the deeper analysis and the broader exploitation of student evaluations. More specifically, we thoroughly examine and evaluate several types of control charts (CCs) aiming at identifying the most suitable among them per case, using as comparison criteria various statistical properties of CCs, such as the average run length (ARL), the median run length (MRL) and the standard deviation of run lengths (SDRL). The ultimate goal of our research is to provide decision makers in HE institutions with a useful and reliable tool for not only monitoring the highly significant teaching process but also identifying the effective and ineffective faculty members' teaching performance. Through this, the former can have an idea about the overall quality of their Institution.

## **KEYWORDS**

Quality management; SPC; Control chart; ARL; MRL; SDRL; Simulation; Monte Carlo.

Although in practice, the distribution of student ratings usually presents a clear negative skew, just like the one depicted in Figure 3, in our numerical investigation we also examine an imaginary, absolutely symmetric distribution of student evaluations (Figure 4), to find out whether our results on the performance of the examined CCs are affected by the skew of the adopted ratings' distribution.

**Figure 4 about here**

## 5. Numerical investigation

The ultimate goal of our research is to compare  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA CCs and, more specifically, 11 variations of these CCs, considering that they are used for monitoring the average student rating for the particular question we mention in Section 4.4. The performance of all these CCs is assessed and compared in terms of their  $ARL_j$ ,  $MRL_j$  and  $SDRL_j$  values for  $j = 0$  or  $1$  (Tables 2 to 17). The general findings are summarized in the following subsections.

### 5.1 Considering the real data distribution of student evaluations

- Focusing on the in control teaching processes, i.e. when  $\delta = 0$ , in case  $ARL_0$  is kept **identical** for all compared CCs (Tables 2 to 5):
  - We notice that it is always  $ARL_0 > MRL_0$ . This is normal as the RL distribution is right-skewed (Figure 2) and in positive-skewed distributions the average is always larger than the median.
  - In terms of  $SDRL_0$ :
    - i) EWMA CC with  $\lambda = 0.1$  has the best performance among the examined CCs, when fixed  $ARL_0$  is equal to 30 (Figure 5, for  $\delta = 0$ ) or 50. Then comes CUSUM CC with  $K = 0.4$ .
    - ii) When  $ARL_0 = 100$ , CUSUM CC with  $K = 0.5$  performs better than the other CCs. Then almost all EWMA CCs come.
    - iii) When  $ARL_0 = 370$ , EWMA CC with  $\lambda = 0.5$  has the best performance. Then comes again CUSUM CC with  $K = 0.4$ .
  - We notice that keeping  $ARL_0$  values identical for all examined CCs, does not ensure identical  $MRL_0$  values: the differences among the latter are sometimes significant, especially for larger fixed  $ARL_0$  values (i.e. 100 and 370)

**Figure 5 about here**

- Focusing on the out of control teaching processes, i.e. when  $\delta \neq 0$ , in case  $ARL_0$  values are kept **identical** for all examined CCs (Tables 2 to 5):
  - Not surprisingly, we see that it is always  $ARL_1 > MRL_1$ . Moreover, the increase of  $\delta$  results in the decrease of both  $ARL_1$  and  $MRL_1$  values, for any type of CC. Additionally, the difference  $ARL_1 - MRL_1$  decreases as  $\delta$  increases, which means that progressively  $ARL_1$  and  $MRL_1$  values converge, independently of the examined CC. This is another indication that the skew of the RL distribution is reduced with the magnitude of the process mean shift (Figure 2).

- Concentrating on the MA CC, we observe that the increase of  $w$  improves the performance of the particular CC without any exceptions: mainly the  $ARL_1$  and  $MRL_1$ , and most of the times the  $SDRL_1$  values decrease when  $w$  increases.
- In Tables 2 to 5 we do not present teaching processes with  $\delta > 0.3524$ , because in our experimentation we have noticed that for such  $\delta$  values  $ARL_1$  tends to 1. Therefore, there is no performance differentiation in the examined CCs: for large shifts, all CCs can identify the assignable cause almost immediately.
- Not only a similar but an even more intensive tendency is ascertained as per the  $MRL_1$  values: for even lower  $\delta$  values,  $MRL_1$  of all examined CCs become equal to 1, which means that all CCs can identify the assignable cause at the first sample (i.e. at the first semester that student evaluation takes place) after the appearance of the assignable cause.
- The comparison of the 11 types of CCs that we have selected in this study reveals the following:
  - i) We notice that for all  $\delta$  values but 0.0201, the MA CC with  $w = 5$  has the best  $ARL_1$  performance no matter which the fixed  $ARL_0$  value is.
  - ii) EWMA CC with  $\lambda = 0.1$  outperforms the other examined CCs regarding  $ARL_1$ , only for  $\delta = 0.0201$  and all fixed  $ARL_0$  values but 30.
  - iii) For the larger shifts that we study (i.e.  $0.151 \leq \delta \leq 0.3524$ ), again the MA CC with  $w = 5$  has the lowest  $SDRL_1$  value, no matter which the fixed  $ARL_0$  value is<sup>1</sup>.
  - iv) For low  $\delta$  values (i.e.  $0.0201 \leq \delta \leq 0.1208$ ), EWMA CC with  $\lambda = 0.1$  presents the lowest  $SDRL_1$  value (e.g. Figure 5).
- As per the  $MRL_1$  values, the advantage of the MA CC with  $w = 5$  is verified for all  $\delta$  and fixed  $ARL_0$  values<sup>2</sup>.
- The fact that we examine low  $\delta$  values, results in a small probability of signal (i.e.  $1 - \beta$ ) of the  $\bar{X}$  CC (and to some extent for the rest of the examined CCs). Considering also
  - i) that the RL is geometrically distributed and
  - ii) the formulas determining the average and the standard deviation of the geometric distribution,
 then it is not surprising that we notice no significant difference between the ARL and their corresponding SDRL values, for the  $\bar{X}$  CC (and to some extent for the rest of the examined CCs), for any  $\delta$  value. This observation verifies the relevant analysis of Quesenberry (1992).
- Focusing on the in control cases (i.e.  $\delta = 0$ ), when we keep  **$MRL_0$**  values **identical** for all compared CCs (Tables 6 to 9):
  - Apparently, we see that it is always  $ARL_0 > MRL_0$ .
  - As per the best (i.e. minimum)  $SDRL_0$  values of the examined CCs, in this case we come to different conclusions regarding the ones we arrive at when we keep  $ARL_0$  fixed:
    - i) CUSUM CC with  $K = 0.4$  performs better, when fixed  $MRL_0$  is 30,
    - ii) EWMA CC with  $\lambda = 0.25$  has the best performance, when fixed  $MRL_0$  is 50,

<sup>1</sup> We identify only one exception: for  $\delta = 0.151$  and  $ARL_0 = 370$  the minimum  $SDRL_1$  is noticed for EWMA CC with  $\lambda = 0.1$ .

<sup>2</sup> We identify only one exception: for  $ARL_0 = 370$  and  $\delta = 0.0201$  the minimum  $MRL_1$  is noticed for EWMA CC with  $\lambda = 0.1$

- iii)  $\bar{X}$  and EWMA CC with  $\lambda = 0.5$  have the best performance, when fixed  $MRL_0$  is 100 and
- iv) EWMA CC with  $\lambda = 0.75$  performs better, when fixed  $MRL_0$  is 370.

Evidently, no single conclusion can be drawn.

- Similarly to what happens when we try to keep  $ARL_0$  values identical for all examined CCs, we notice that fixing  $MRL_0$  values does not ensure identical  $ARL_0$  ones. This becomes obvious in several cases, for example when fixed  $MRL_0 = 100$ , the maximum  $ARL_0$  value is 170.056 and the minimum 140.787 (Figure 6, for  $\delta = 0$ ).

#### Figure 6 about here

- Focusing on the out of control teaching processes (i.e.  $\delta \neq 0$ ), in case  $MRL_0$  values are kept **identical** for all examined CCs (Tables 6 to 9):
  - We verify our previous findings that the increase of  $\delta$ 
    - i) results in the decrease of both  $ARL_1$  (Figure 6) and  $MRL_1$  values, for any studied CC and
    - ii) make  $ARL_1$  and  $MRL_1$  values to converge.
  - Focusing on the MA CC, we observe again that the higher the value of  $w$  the better the performance of the particular CC, for all  $\delta$  values except from the case where  $\delta = 0.020$ . This means that the  $ARL_1$ ,  $MRL_1$  and  $SDRL_1$  values decrease as  $w$  increases.
  - In this group of Tables (i.e. 6 to 9) we still do not present teaching processes with  $\delta > 0.3524$  because we have found out that for such  $\delta$  values every  $ARL_1$  becomes almost equal to 1. Therefore, all examined CCs become equivalently efficient in identifying any assignable cause almost instantaneously.
  - Similarly, an even more intensive tendency is recognized for the  $MRL_1$  values too: for even lower  $\delta$  values than 0.3524,  $MRL_1$  becomes equal to 1, which means that in practice all CCs identify the assignable cause at the first sample (i.e student evaluation) after its appearance.
  - Comparing the 11 CCs of our study regarding their performance, we notice that
    - i) for shifts with  $\delta = 0.0201$  (as well as  $\delta = 0.0705$  and fixed  $MRL_0 = 370$ ), EWMA CC with  $\lambda = 0.1$  performs better (i.e. has the minimum  $ARL_1$  value) for any fixed  $MRL_0$  value (Figure 6),
    - ii) for  $\delta \geq 0.0705$ , the MA CC with  $w = 5$  has the best  $ARL_1$  performance (Figure 6).
  - Another conclusion arising from Tables 6 to 9 is that for low  $\delta$  values, the EWMA  $SDRL_1$  values for  $\lambda = 0.1$  are the smallest among all studied CCs, for all fixed  $MRL_0$  values.
  - As per the  $MRL_1$  values, the advantage of the MA CC with  $w = 5$  is verified for all  $\delta$  and fixed  $MRL_0$  values, apart from the case of  $\delta = 0.0201$  and fixed  $MRL_0 = 100$  or 370. In these cases EWMA CC with  $\lambda = 0.1$  outperforms the rest CCs.
  - Even when we keep the  $MRL_0$  values identical instead of the  $ARL_0$  ones, in order to compare our CCs, we verify that there is no significant difference between the ARL and the corresponding  $SDRL$  values, for the  $\bar{X}$  CC (and to some extent for the rest of the examined CCs).

## 5.2 Considering the imaginary symmetric distribution of student evaluations

- Focusing on the in control teaching processes (i.e.  $\delta = 0$ ), in case  $ARL_0$  is kept **identical** for all examined CCs (Tables 10 to 13):
  - Not unexpectedly, we still notice that the  $ARL_0$  values are always larger than the respective  $MRL_0$  values, independently of the examined CC.
  - In terms of  $SDRL_0$  values, the optimal CC is different from case to case:
    - i) CUSUM CC with  $K = 0.4$  has the best performance among the examined CCs, when fixed  $ARL_0$  is equal to 30 or 50,
    - ii)  $\bar{X}$  CC outperforms the other CCs for large fixed  $ARL_0$  values, i.e. 100 or 370.
  - Again, keeping fixed  $ARL_0$  values identical for all CCs does not mean constant  $MRL_0$  values for the examined CCs.
- Focusing on the out of control teaching processes (i.e.  $\delta \neq 0$ ), in case we keep  $ARL_0$  values **identical** for all compared CCs (Tables 10 to 13):
  - Once again, the increase of  $\delta$  results in the decrease of i) the  $ARL_1$  and  $MRL_1$  values, as well as ii) the difference  $ARL_1 - MRL_1$ , for all CCs.
  - Concentrating on the MA CCs, we observe again that the increase of  $w$  improves the performance of the particular CC type without significant exceptions:  $ARL_1$ ,  $MRL_1$  and in the vast majority of cases  $SDRL_1$  values decrease when  $w$  increases.
  - For large  $\delta$  values, the  $ARL_1$  values of all examined CCs tend to 1. Much more intensively,  $MRL_1$  values become equal to 1 for even lower  $\delta$  values. This means that for such shifts all CCs become equivalently efficient and can identify immediately any assignable cause.
  - Comparing the  $ARL_1$  performance of the 11 studied CCs, we notice that
    - i) Almost always (i.e. for shifts of magnitude  $\delta \geq 0.0705$ ), the MA CC with  $w = 5$  has the best  $ARL_1$  performance for all examined fixed  $ARL_0$  values.
    - ii) EWMA CC with  $\lambda = 0.1$  outperforms the other CCs only when  $\delta = 0.0201$  and fixed  $ARL_0$  is equal to 50, 100 and 370.
  - Comparing the  $SDRL_1$  performance of the 11 CCs, we notice that for shifts of small magnitude (namely for  $\delta \leq 0.1208$  and, when fixed  $ARL_0 = 370$ , for  $\delta = 0.151$  too), EWMA CC with  $\lambda = 0.1$  outperforms the other examined CCs. For larger  $\delta$  values the MA CC with  $w = 5$  has the best  $SDRL_1$  performance.
  - As per the  $MRL_1$  values, the advantage of the MA CC with  $w = 5$  is verified for all  $\delta$  and fixed  $ARL_0$  values.
- Focusing on the in control cases (i.e.  $\delta = 0$ ), when  $MRL_0$  is **fixed** for all examined CCs (Tables 14 to 17):
  - Not surprisingly, we see that it is always  $ARL_0 > MRL_0$ .
  - As per the minimum  $SDRL_0$  values of the examined CCs, we come to the following conclusions:
    - i) CUSUM CC with  $K = 0.5$  performs better, when fixed  $MRL_0$  is 30,
    - ii) EWMA CC with  $\lambda = 0.75$  has the best performance, when fixed  $MRL_0$  is 50 and
    - iii)  $\bar{X}$  CC performs better, when fixed  $MRL_0$  is 100 or 370.
  - We conclude one more time that keeping  $MRL_0$  values identical does not ensure constant  $ARL_0$  values.

- Focusing on the out of control teaching processes (i.e.  $\delta \neq 0$ ), in case we keep **MRL<sub>0</sub>** values **identical** for all compared CCs (Tables 14 to 17):
  - We verify once again that the increase of  $\delta$  results in the decrease of i) both ARL<sub>1</sub> and MRL<sub>1</sub> values, for any CC type as well as ii) the difference between them, i.e. ARL<sub>1</sub> - MRL<sub>1</sub>.
  - Focusing on the MA CC, we observe that the higher the  $w$ , the better the performance of the particular CC is: exceptions are noticed only for  $\delta = 0.0201$ . This means that mainly the ARL<sub>1</sub>, MRL<sub>1</sub> and SDRL<sub>1</sub> values decrease as  $w$  increases.
  - For large  $\delta$  values the ARL<sub>1</sub> values tend to 1, while for even lower  $\delta$  values the MRL<sub>1</sub> values become equal to 1. This means that for such shifts all CCs identify in practice the assignable cause at the first student evaluation after its appearance.
  - Comparing the ARL<sub>1</sub> values of the 11 studied CCs, we notice that
    - i) With only one exception, whenever  $\delta \geq 0.0705$ , the MA CC with  $w = 5$  has the best ARL<sub>1</sub> performance for all examined fixed MRL<sub>0</sub> values.
    - ii) EWMA CC with  $\lambda = 0.1$  outperforms the other CCs only when  $\delta = 0.0201$  (for all fixed MRL<sub>0</sub> values) and  $\delta = 0.0705$  (for fixed MRL<sub>0</sub> equal to 370).
  - In this case, the comparison of the SDRL<sub>1</sub> performance of the 11 compared CCs of our study leads to exactly the same conclusions as in the respective case of Section 5.1: for low  $\delta$  values EWMA CC with  $\lambda = 0.1$  outperforms the other examined CCs, while for larger  $\delta$  values the MA CC with  $w = 5$  has the best SDRL<sub>1</sub> performance.
  - As per the MRL<sub>1</sub> values, the advantage of the MA CC with  $w = 5$  is verified for almost any  $\delta$  and fixed MRL<sub>0</sub> value, apart from the case of  $\delta = 0.0201$  and fixed MRL<sub>0</sub> = 100 or 370, when EWMA CC with  $\lambda = 0.1$  outperforms the rest CCs.

**Table 2.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $ARL_0 = 30$

$\delta$	$\bar{X}$ $L = 2.128$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
		$L = 1.559$ $\lambda = 0.1$	$L = 1.891$ $\lambda = 0.25$	$L = 2.052$ $\lambda = 0.50$	$L = 2.109$ $\lambda = 0.75$	$H = 2.798$ $K = 0.4$	$H = 2.402$ $K = 0.5$	$L = 2.117$ $w = 2$	$L = 2.100$ $w = 3$	$L = 2.094$ $w = 4$	$L = 2.099$ $w = 5$	
<b>0</b>	MRL <sub>0</sub>	21	22	21	21	21	22	22	20	17	15	11
	ARL <sub>0</sub>	30.108	30.012	30.011	29.993	29.991	30.002	30.002	29.851	30.028	29.992	30.002
	SDRL <sub>0</sub>	30.202	26.279	27.835	28.675	29.821	26.859	27.365	32.603	36.017	39.427	43.268
<b>0.0201</b>	MRL <sub>1</sub>	17	15	16	16	17	16	16	16	13	10	7
	ARL <sub>1</sub>	25.287	20.231	21.246	22.708	23.925	22.018	22.539	23.878	22.086	20.675	19.830
	SDRL <sub>1</sub>	24.985	16.935	19.033	21.922	23.245	18.944	20.050	25.249	26.670	27.055	27.997
<b>0.0705</b>	MRL <sub>1</sub>	7	5	5	5	6	5	5	4	3	2	1
	ARL <sub>1</sub>	9.515	5.905	5.822	6.350	7.539	6.304	6.322	6.457	5.162	4.389	3.905
	SDRL <sub>1</sub>	8.978	3.240	3.825	5.020	6.690	3.9178	4.288	6.578	5.650	5.150	4.788
<b>0.1208</b>	MRL <sub>1</sub>	3	3	3	3	3	3	3	2	1	1	1
	ARL <sub>1</sub>	3.875	3.366	3.057	2.976	3.232	3.337	3.206	2.561	2.087	1.833	1.669
	SDRL <sub>1</sub>	3.340	1.358	1.495	1.835	2.404	1.533	1.611	2.243	1.854	1.628	1.449
<b>0.1510</b>	MRL <sub>1</sub>	2	3	2	2	2	2	2	1	1	1	1
	ARL <sub>1</sub>	2.566	2.709	2.394	2.207	2.265	2.615	2.479	1.776	1.502	1.368	1.291
	SDRL <sub>1</sub>	1.998	0.957	1.025	1.191	1.477	1.045	1.082	1.326	1.062	0.895	0.778
<b>0.2416</b>	MRL <sub>1</sub>	1	2	1	1	1	2	1	1	1	1	1
	ARL <sub>1</sub>	1.252	1.795	1.487	1.300	1.246	1.642	1.510	1.084	1.044	1.030	1.022
	SDRL <sub>1</sub>	0.563	0.543	0.558	0.507	0.501	0.578	0.570	0.327	0.230	0.185	0.158
<b>0.2719</b>	MRL <sub>1</sub>	1	2	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.129	1.619	1.328	1.173	1.131	1.463	1.346	1.037	1.018	1.012	1.009
	SDRL <sub>1</sub>	0.381	0.526	0.488	0.394	0.361	0.528	0.498	0.205	0.140	0.113	0.096
<b>0.3121</b>	MRL <sub>1</sub>	1	1	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.047	1.403	1.168	1.074	1.051	1.266	1.179	1.010	1.005	1.003	1.002
	SDRL <sub>1</sub>	0.223	0.496	0.376	0.264	0.224	0.446	0.386	0.103	0.068	0.052	0.043
<b>0.3524</b>	MRL <sub>1</sub>	1	1	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.015	1.231	1.073	1.027	1.017	1.132	1.079	1.003	1.001	1.001	1.000
	SDRL <sub>1</sub>	0.124	0.422	0.260	0.162	0.129	0.339	0.270	0.051	0.034	0.026	0.021

**Table 3.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $ARL_0 = 50$

$\delta$	$\bar{X}$ $L = 2.326$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
		$L = 1.820$ $\lambda = 0.1$	$L = 2.124$ $\lambda = 0.25$	$L = 2.266$ $\lambda = 0.50$	$L = 2.303$ $\lambda = 0.75$	$H = 3.347$ $K = 0.4$	$H = 2.857$ $K = 0.5$	$L = 2.311$ $w = 2$	$L = 2.302$ $w = 3$	$L = 2.301$ $w = 4$	$L = 2.289$ $w = 5$	
0	MRL <sub>0</sub>	34	37	36	35	35	36	35	34	32	28	24
	ARL <sub>0</sub>	49.980	49.997	50.022	49.997	49.955	50.021	49.971	49.903	50.021	49.972	50.023
	SDRL <sub>0</sub>	50.833	45.349	47.139	48.903	49.635	46.500	46.654	52.935	56.749	65.130	68.427
0.0201	MRL <sub>1</sub>	28	21	23	25	26	23	24	25	23	18	13
	ARL <sub>1</sub>	40.727	27.982	31.343	34.789	37.155	31.343	32.557	37.070	35.119	33.372	29.998
	SDRL <sub>1</sub>	40.607	23.515	28.073	32.520	36.292	27.616	29.478	38.892	40.121	42.549	41.135
0.0705	MRL <sub>1</sub>	10	6	6	6	7	6	6	6	4	2	2
	ARL <sub>1</sub>	13.710	7.036	6.977	8.096	10.123	7.484	7.485	8.575	6.692	5.552	4.747
	SDRL <sub>1</sub>	13.018	3.699	4.632	6.563	9.169	4.493	4.928	8.892	7.416	6.594	5.911
0.1208	MRL <sub>1</sub>	4	4	3	3	3	4	3	2	1	1	1
	ARL <sub>1</sub>	5.070	3.897	3.478	3.462	3.894	3.850	3.677	3.055	2.395	2.060	1.837
	SDRL <sub>1</sub>	4.516	1.513	1.678	2.189	3.005	1.670	1.746	2.799	2.246	1.952	1.714
0.1510	MRL <sub>1</sub>	3	3	2	2	2	3	3	1	1	1	1
	ARL <sub>1</sub>	3.173	3.094	2.683	2.507	2.625	2.999	2.816	2.008	1.643	1.465	1.356
	SDRL <sub>1</sub>	2.628	1.050	1.114	1.369	1.777	1.122	1.154	1.595	1.261	1.060	0.901
0.2416	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.355	2.034	1.655	1.406	1.332	1.892	1.727	1.114	1.059	1.038	1.028
	SDRL <sub>1</sub>	0.692	0.521	0.588	0.575	0.585	0.570	0.594	0.389	0.272	0.213	0.180
0.2719	MRL <sub>1</sub>	1	2	1	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.186	1.858	1.471	1.248	1.184	1.700	1.535	1.050	1.025	1.015	1.011
	SDRL <sub>1</sub>	0.470	0.486	0.533	0.459	0.427	0.537	0.544	0.243	0.166	0.129	0.110
0.3121	MRL <sub>1</sub>	1	2	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.072	1.648	1.271	1.116	1.077	1.474	1.322	1.015	1.006	1.004	1.002
	SDRL <sub>1</sub>	0.279	0.500	0.450	0.324	0.273	0.512	0.474	0.127	0.081	0.062	0.050
0.3524	MRL <sub>1</sub>	1	1	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.025	1.447	1.135	1.046	1.027	1.286	1.169	1.004	1.002	1.001	1.001
	SDRL <sub>1</sub>	0.159	0.500	0.342	0.209	0.165	0.454	0.376	0.063	0.041	0.032	0.025



**Table 4.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $ARL_0 = 100$

$\delta$	$\bar{X}$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
	$L = 2.576$	$L = 2.15$ $\lambda = 0.1$	$L = 2.406$ $\lambda = 0.25$	$L = 2.535$ $\lambda = 0.50$	$L = 2.582$ $\lambda = 0.75$	$H = 4.106$ $K = 0.4$	$H = 3.510$ $K = 0.5$	$L = 2.557$ $w = 2$	$L = 2.551$ $w = 3$	$L = 2.512$ $w = 4$	$L = 2.548$ $w = 5$	
0	MRL <sub>0</sub>	71	68	69	66.5	71	67	67.5	67	64.500	58	56
	ARL <sub>0</sub>	100.461	99.995	99.993	99.959	100.159	99.993	99.993	100.086	99.979	100.294	99.993
	SDRL <sub>0</sub>	100.292	96.715	100.159	96.146	96.388	98.048	95.656	104.100	119.046	122.505	130.079
0.0201	MRL <sub>1</sub>	56	31	36	45	52	35	40	46	40	34	28
	ARL <sub>1</sub>	81.024	42.170	50.497	62.49	73.843	48.937	54.455	69.430	63.361	59.690	54.251
	SDRL <sub>1</sub>	83.562	35.414	45.715	58.752	71.774	43.0634	49.514	73.110	71.660	73.427	72.242
0.0705	MRL <sub>1</sub>	16	8	7	8	12	8	8	9	6	5	2
	ARL <sub>1</sub>	22.507	8.642	8.760	11.249	16.011	9.080	9.210	12.864	9.420	7.624	6.402
	SDRL <sub>1</sub>	21.293	4.466	5.888	9.553	14.827	5.177	5.829	13.518	10.595	9.081	8.119
0.1208	MRL <sub>1</sub>	5	4	4	4	4	4	4	3	1	1	1
	ARL <sub>1</sub>	7.416	4.604	4.064	4.248	5.259	4.560	4.356	3.921	2.909	2.414	2.108
	SDRL <sub>1</sub>	6.831	1.714	1.957	2.767	4.264	1.847	1.932	3.775	2.884	2.426	2.129
0.1510	MRL <sub>1</sub>	3	3	3	3	3	3	3	1	1	1	1
	ARL <sub>1</sub>	4.286	3.619	3.070	2.959	3.319	3.514	3.301	2.398	1.867	1.613	1.465
	SDRL <sub>1</sub>	3.755	1.175	1.255	1.641	2.384	1.235	1.261	2.047	1.564	1.293	1.100
0.2416	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.536	2.303	1.861	1.563	1.488	2.194	2.021	1.162	1.081	1.052	1.037
	SDRL <sub>1</sub>	0.906	0.556	0.601	0.653	0.717	0.565	0.584	0.479	0.330	0.253	0.210
0.2719	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.290	2.100	1.662	1.371	1.287	1.992	1.819	1.075	1.035	1.022	1.015
	SDRL <sub>1</sub>	0.613	0.455	0.554	0.536	0.531	0.489	0.535	0.304	0.202	0.154	0.127
0.3121	MRL <sub>1</sub>	1	2	1	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.119	1.907	1.430	1.188	1.130	1.778	1.585	1.023	1.010	1.006	1.004
	SDRL <sub>1</sub>	0.367	0.411	0.509	0.401	0.353	0.475	0.520	0.159	0.100	0.075	0.061
0.3524	MRL <sub>1</sub>	1	2	1	1	1	2	1	1	1	1	1
	ARL <sub>1</sub>	1.043	1.741	1.251	1.083	1.051	1.582	1.383	1.006	1.002	1.001	1.001
	SDRL <sub>1</sub>	0.212	0.456	0.435	0.277	0.223	0.502	0.490	0.082	0.050	0.037	0.030

**Table 5.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $ARL_0 = 370$

$\delta$	$\bar{X}$ $L = 3.040$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
		$L = 2.720$ $\lambda = 0.1$	$L = 2.918$ $\lambda = 0.25$	$L = 3.010$ $\lambda = 0.5$	$L = 3.014$ $\lambda = 0.75$	$H = 5.651$ $K = 0.4$	$H = 4.735$ $K = 0.5$	$L = 2.998$ $w = 2$	$L = 2.977$ $w = 3$	$L = 2.972$ $w = 4$	$L = 2.987$ $w = 5$	
0	MRL <sub>0</sub>	252	230.500	260.500	256	259.500	230	241	242	251	229.500	186
	ARL <sub>0</sub>	369.425	369.670	369.670	369.511	369.511	370.352	370.352	369.511	371.590	369.670	371.721
	SDRL <sub>0</sub>	368.302	399.982	374.060	351.240	364.125	352.587	367.063	384.575	393.023	408.593	490.250
0.0201	MRL <sub>1</sub>	233	71	106.500	144	190.500	80	91	154	142	111.500	103
	ARL <sub>1</sub>	334.378	93.438	141.613	216.874	261.725	106.587	122.674	237.480	211.372	177.899	168.034
	SDRL <sub>1</sub>	317.465	79.424	130.164	226.908	247.599	93.773	110.143	241.971	216.289	199.356	211.158
0.0705	MRL <sub>1</sub>	48	11	11	16	27	11	11	21	12	8	6
	ARL <sub>1</sub>	66.150	12.085	13.775	22.271	37.074	12.434	12.496	30.088	18.966	14.114	11.595
	SDRL <sub>1</sub>	63.847	6.146	9.854	19.805	35.085	6.449	7.472	30.386	20.905	17.060	14.867
0.1208	MRL <sub>1</sub>	12	6	5	5	7	6	5	5	2	1	1
	ARL <sub>1</sub>	17.086	5.967	5.366	6.414	9.358	6.002	5.609	6.746	4.353	3.338	2.817
	SDRL <sub>1</sub>	16.761	2.109	2.638	4.547	8.129	2.153	2.249	6.832	4.619	3.638	3.124
0.1510	MRL <sub>1</sub>	6	4	4	3	4	4	4	3	1	1	1
	ARL <sub>1</sub>	8.555	4.593	3.882	4.034	5.123	4.586	4.208	3.583	2.465	1.984	1.736
	SDRL <sub>1</sub>	8.067	1.394	1.594	2.379	4.075	1.421	1.459	3.406	2.317	1.832	1.544
0.2416	MRL <sub>1</sub>	2	3	2	2	2	3	2	1	1	1	1
	ARL <sub>1</sub>	2.135	2.809	2.228	1.908	1.842	2.769	2.500	1.304	1.140	1.084	1.060
	SDRL <sub>1</sub>	1.570	0.672	0.635	0.776	0.985	0.677	0.644	0.706	0.457	0.341	0.276
0.2719	MRL <sub>1</sub>	1	2	2	2	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.631	2.514	1.996	1.641	1.525	2.477	2.243	1.145	1.062	1.036	1.025
	SDRL <sub>1</sub>	1.010	0.579	0.538	0.644	0.722	0.576	0.521	0.449	0.279	0.206	0.168
0.3121	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.274	2.218	1.755	1.379	1.261	2.190	2.011	1.050	1.019	1.010	1.007
	SDRL <sub>1</sub>	0.594	0.442	0.504	0.521	0.498	0.433	0.416	0.242	0.142	0.102	0.082
0.3524	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.112	2.047	1.545	1.203	1.118	2.020	1.843	1.015	1.005	1.003	1.002
	SDRL <sub>1</sub>	0.351	0.315	0.510	0.410	0.335	0.323	0.417	0.128	0.071	0.051	0.041

**Table 6.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $MRL_0 = 30$

$\delta$	$\bar{X}$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
	$L = 2.277$	$L = 1.729$ $\lambda = 0.1$	$L = 2.056$ $\lambda = 0.25$	$L = 2.205$ $\lambda = 0.50$	$L = 2.250$ $\lambda = 0.75$	$H = 3.150$ $K = 0.4$	$H = 2.700$ $K = 0.5$	$L = 2.265$ $w = 2$	$L = 2.281$ $w = 3$	$L = 2.334$ $w = 4$	$L = 2.364$ $w = 5$	
0	MRL <sub>0</sub>	30	30	30	30	30	30	30	30	30	30	
	ARL <sub>0</sub>	43.861	41.525	43.138	43.044	43.123	41.356	42.283	44.067	47.457	54.881	60.164
	SDRL <sub>0</sub>	44.405	37.521	40.220	41.997	43.279	37.183	39.201	46.770	54.855	70.336	81.383
0.0201	MRL <sub>1</sub>	25	19	21	22	23	21	21	22	21	19	16
	ARL <sub>1</sub>	36.187	25.076	28.188	30.936	32.825	27.827	28.830	33.216	33.173	35.811	35.443
	SDRL <sub>1</sub>	35.445	21.331	25.511	29.531	32.024	24.535	26.172	35.281	37.936	45.842	47.189
0.0705	MRL <sub>1</sub>	9	6	5	6	7	6	6	5	4	3	2
	ARL <sub>1</sub>	12.546	6.642	6.605	7.538	9.290	7.060	7.111	8.018	6.519	5.800	5.165
	SDRL <sub>1</sub>	11.885	3.527	4.356	6.084	8.342	4.300	4.723	8.303	7.223	6.921	6.476
0.1208	MRL <sub>1</sub>	3	3	3	3	3	3	3	2	1	1	1
	ARL <sub>1</sub>	4.720	3.706	3.357	3.322	3.696	3.673	3.517	2.927	2.358	2.101	1.907
	SDRL <sub>1</sub>	4.184	1.461	1.626	2.087	2.820	1.626	1.700	2.656	2.205	2.008	1.824
0.1510	MRL <sub>1</sub>	2	3	2	2	2	3	2	1	1	1	1
	ARL <sub>1</sub>	3.009	2.962	2.596	2.420	2.516	2.858	2.695	1.949	1.626	1.481	1.384
	SDRL <sub>1</sub>	2.455	1.015	1.083	1.318	1.689	1.094	1.130	1.528	1.236	1.088	0.954
0.2416	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.327	1.954	1.604	1.372	1.306	1.805	1.651	1.106	1.057	1.040	1.031
	SDRL <sub>1</sub>	0.658	0.527	0.582	0.555	0.561	0.575	0.591	0.374	0.267	0.218	0.188
0.2719	MRL <sub>1</sub>	1	2	1	1	1	2	1	1	1	1	1
	ARL <sub>1</sub>	1.170	1.778	1.428	1.224	1.168	1.615	1.467	1.047	1.024	1.016	1.012
	SDRL <sub>1</sub>	0.446	0.505	0.523	0.440	0.408	0.541	0.534	0.234	0.163	0.132	0.115
0.3121	MRL <sub>1</sub>	1	2	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.065	1.562	1.237	1.102	1.069	1.394	1.267	1.014	1.006	1.004	1.003
	SDRL <sub>1</sub>	0.264	0.510	0.430	0.307	0.260	0.497	0.448	0.121	0.080	0.064	0.053
0.3524	MRL <sub>1</sub>	1	1	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.022	1.366	1.113	1.039	1.024	1.222	1.132	1.004	1.002	1.001	1.001
	SDRL <sub>1</sub>	0.149	0.484	0.318	0.195	0.155	0.417	0.340	0.061	0.040	0.032	0.027

**Table 7.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $MRL_0 = 50$

$\delta$	$\bar{X}$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
	$L = 2.473$	$L = 1.999$ $\lambda = 0.1$	$L = 2.255$ $\lambda = 0.25$	$L = 2.412$ $\lambda = 0.50$	$L = 2.458$ $\lambda = 0.75$	$H = 3.726$ $K = 0.4$	$H = 3.162$ $K = 0.5$	$L = 2.455$ $w = 2$	$L = 2.471$ $w = 3$	$L = 2.510$ $w = 4$	$L = 2.504$ $w = 5$	
0	MRL <sub>0</sub>	50	50	50	50	50	50	50	50	50	50	
	ARL <sub>0</sub>	73.230	71.169	68.866	72.871	71.861	70.817	74.933	78.609	89.922	87.560	
	SDRL <sub>0</sub>	74.412	67.585	64.883	72.014	69.232	65.586	78.589	88.735	111.452	110.819	
0.0201	MRL <sub>1</sub>	42	27	29	35	38	29	31	36	32	31	25
	ARL <sub>1</sub>	60.304	35.068	39.163	47.096	53.525	39.486	41.416	53.926	52.430	54.754	49.326
	SDRL <sub>1</sub>	59.974	29.291	34.941	43.357	52.717	34.168	36.970	57.597	59.571	68.278	66.374
0.0705	MRL <sub>1</sub>	13	7	6	7	9	7	7	7	5	4	2
	ARL <sub>1</sub>	18.286	7.871	7.758	9.636	12.974	8.285	8.288	10.841	8.447	7.230	6.065
	SDRL <sub>1</sub>	17.282	4.100	5.169	8.027	11.797	4.841	5.355	11.278	9.468	8.614	7.645
0.1208	MRL <sub>1</sub>	5	4	3	3	3	4	4	3	1	1	1
	ARL <sub>1</sub>	6.318	4.275	3.734	3.867	4.575	4.213	3.995	3.520	2.724	2.345	2.052
	SDRL <sub>1</sub>	5.720	1.629	1.792	2.486	3.649	1.758	1.843	3.325	2.652	2.341	2.042
0.1510	MRL <sub>1</sub>	3	3	3	2	2	3	3	1	1	1	1
	ARL <sub>1</sub>	3.765	3.368	2.859	2.740	2.981	3.254	3.045	2.222	1.785	1.587	1.445
	SDRL <sub>1</sub>	3.248	1.119	1.177	1.503	2.076	1.183	1.203	1.846	1.455	1.252	1.064
0.2416	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.455	2.182	1.753	1.486	1.412	2.049	1.869	1.140	1.073	1.049	1.035
	SDRL <sub>1</sub>	0.813	0.533	0.596	0.617	0.656	0.560	0.591	0.439	0.309	0.246	0.204
0.2719	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.242	1.996	1.558	1.311	1.236	1.855	1.670	1.064	1.031	1.020	1.014
	SDRL <sub>1</sub>	0.548	0.459	0.547	0.501	0.482	0.513	0.549	0.278	0.189	0.150	0.124
0.3121	MRL <sub>1</sub>	1	2	1	1	1	2	1	1	1	1	1
	ARL <sub>1</sub>	1.098	1.798	1.342	1.152	1.103	1.631	1.440	1.019	1.008	1.005	1.003
	SDRL <sub>1</sub>	0.328	0.456	0.482	0.366	0.316	0.511	0.509	0.145	0.094	0.073	0.059
0.3524	MRL <sub>1</sub>	1	2	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.035	1.612	1.183	1.064	1.039	1.427	1.258	1.005	1.002	1.001	1.001
	SDRL <sub>1</sub>	0.189	0.495	0.387	0.245	0.196	0.499	0.440	0.073	0.047	0.036	0.030

**Table 8.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $MRL_0 = 100$

$\delta$	$\bar{X}$	EWMA					CUSUM $\bar{X}$		Moving Average (MA)			
	$L = 2.694$	$L = 2.323$ $\lambda = 0.1$	$L = 2.576$ $\lambda = 0.25$	$L = 2.674$ $\lambda = 0.50$	$L = 2.709$ $\lambda = 0.75$	$H = 4.583$ $K = 0.4$	$H = 3.836$ $K = 0.5$	$L = 2.693$ $w = 2$	$L = 2.685$ $w = 3$	$L = 2.715$ $w = 4$	$L = 2.744$ $w = 5$	
0	MRL <sub>0</sub>	100	100	100	100	100	100	100	100	101	102	
	ARL <sub>0</sub>	140.787	147.928	148.358	146.353	148.749	144.708	143.668	145.422	148.193	161.507	170.056
	SDRL <sub>0</sub>	140.915	150.143	148.774	141.001	142.795	143.239	143.932	151.137	171.238	188.911	213.547
0.0201	MRL <sub>1</sub>	77.500	40	48	66	76	46	49	65.5	56	54	53
	ARL <sub>1</sub>	111.589	53.410	69.720	88.875	106.707	63.479	68.338	98.405	89.347	91.473	90.644
	SDRL <sub>1</sub>	113.801	46.024	65.439	82.363	102.428	55.333	61.735	105.139	100.751	112.994	113.276
0.0705	MRL <sub>1</sub>	21	8	8	10	14	9	8	11	7	6	3
	ARL <sub>1</sub>	29.047	9.583	10.072	13.496	20.042	10.109	10.088	16.282	11.522	9.578	8.238
	SDRL <sub>1</sub>	27.788	4.897	6.852	11.586	18.687	5.618	6.272	16.853	13.013	11.537	10.397
0.1208	MRL <sub>1</sub>	6	5	4	4	5	5	4	3	2	1	1
	ARL <sub>1</sub>	8.970	5.003	4.457	4.747	6.161	5.001	4.680	4.575	3.270	2.723	2.378
	SDRL <sub>1</sub>	8.438	1.827	2.150	3.165	5.111	1.949	2.017	4.481	3.312	2.832	2.511
0.1510	MRL <sub>1</sub>	4	4	3	3	3	4	3	2	1	1	1
	ARL <sub>1</sub>	5.018	3.899	3.319	3.228	3.751	3.839	3.539	2.685	2.022	1.738	1.569
	SDRL <sub>1</sub>	4.539	1.236	1.361	1.825	2.771	1.286	1.316	2.379	1.767	1.479	1.278
0.2416	MRL <sub>1</sub>	1	2	2	2	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.651	2.448	1.984	1.655	1.575	2.364	2.155	1.197	1.097	1.063	1.046
	SDRL <sub>1</sub>	1.038	0.598	0.606	0.689	0.785	0.594	0.585	0.540	0.367	0.284	0.236
0.2719	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.354	2.216	1.776	1.441	1.347	2.141	1.947	1.092	1.042	1.027	1.019
	SDRL <sub>1</sub>	0.692	0.485	0.552	0.569	0.584	0.487	0.514	0.343	0.222	0.173	0.144
0.3121	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.149	2.011	1.538	1.235	1.161	1.929	1.719	1.029	1.012	1.004	1.005
	SDRL <sub>1</sub>	0.415	0.379	0.523	0.438	0.393	0.419	0.499	0.182	0.113	0.085	0.070
0.3524	MRL <sub>1</sub>	1	2	1	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.056	1.863	1.341	1.110	1.066	1.759	1.512	1.009	1.003	1.002	1.001
	SDRL <sub>1</sub>	0.241	0.390	0.478	0.315	0.254	0.453	0.508	0.095	0.056	0.043	0.035

**Table 9.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $MRL_0 = 370$

$\delta$	$\bar{X}$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
		$L = 3.135$	$L = 2.876$ $\lambda = 0.1$	$L = 3.092$ $\lambda = 0.25$	$L = 3.083$ $\lambda = 0.5$	$L = 3.105$ $\lambda = 0.75$	$H = 6.289$ $K = 0.4$	$H = 5.221$ $K = 0.5$	$L = 3.105$ $w = 2$	$L = 3.080$ $w = 3$	$L = 3.102$ $w = 4$	$L = 3.164$ $w = 5$
0	MRL <sub>0</sub>	370	372	368	368	370	368	372	370	368	368	365
	ARL <sub>0</sub>	553.106	567.108	587.124	500.146	497.633	641.019	598.772	562.229	532.545	562.676	697.387
	SDRL <sub>0</sub>	557.045	608.297	603.07	455.787	450.426	674.173	633.972	558.260	530.813	619.633	853.614
0.0201	MRL <sub>1</sub>	307.500	89	156.500	188	246	97	117	217.5	185.5	168	161.5
	ARL <sub>1</sub>	450.356	121.334	218.295	280.84	362.243	135.29	163.098	326.729	285.654	260.362	268.763
	SDRL <sub>1</sub>	453.058	106.459	209.754	277.49	358.339	120.369	145.438	328.573	297.659	275.085	316.798
0.0705	MRL <sub>1</sub>	64	12	13	19	33	12	12	26	15	10	8
	ARL <sub>1</sub>	86.990	13.278	16.345	26.022	45.309	13.790	13.869	37.825	22.935	17.432	15.167
	SDRL <sub>1</sub>	83.877	6.732	12.164	23.398	42.689	6.933	8.027	38.249	24.785	20.882	19.399
0.1208	MRL <sub>1</sub>	14	6	5	6	8	6	6	5	3	2	1
	ARL <sub>1</sub>	20.634	6.380	5.917	6.963	10.726	6.595	6.108	7.823	4.853	3.742	3.219
	SDRL <sub>1</sub>	20.260	2.226	2.945	5.028	9.407	2.274	2.365	8.033	5.245	4.165	3.666
0.1510	MRL <sub>1</sub>	7	5	4	4	4	5	4	3	1	1	1
	ARL <sub>1</sub>	10.021	4.871	4.187	4.295	5.693	5.015	4.570	4.019	2.660	2.142	1.888
	SDRL <sub>1</sub>	9.508	1.449	1.725	2.595	4.593	1.495	1.512	3.898	2.568	2.045	1.786
0.2416	MRL <sub>1</sub>	2	3	2	2	2	3	3	1	1	1	1
	ARL <sub>1</sub>	2.319	2.959	2.355	1.980	1.937	3.027	2.694	1.354	1.158	1.098	1.071
	SDRL <sub>1</sub>	1.767	0.689	0.665	0.803	1.051	0.701	0.683	0.775	0.493	0.372	0.307
0.2719	MRL <sub>1</sub>	1	3	2	2	1	3	2	1	1	1	1
	ARL <sub>1</sub>	1.735	2.645	2.103	1.698	1.587	2.695	2.400	1.170	1.071	1.042	1.030
	SDRL <sub>1</sub>	1.127	0.603	0.546	0.660	0.767	0.612	0.565	0.494	0.304	0.223	0.186
0.3121	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.324	2.313	1.854	1.422	1.299	2.354	2.131	1.059	1.021	1.012	1.008
	SDRL <sub>1</sub>	0.658	0.487	0.485	0.538	0.531	0.505	0.422	0.266	0.153	0.113	0.092
0.3524	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.134	2.109	1.654	1.235	1.139	2.133	1.965	1.018	1.006	1.003	1.002
	SDRL <sub>1</sub>	0.390	0.346	0.499	0.433	0.361	0.365	0.353	0.141	0.078	0.056	0.047

**Table 10.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $ARL_0 = 30$

$\delta$	$\bar{X}$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
	$L = 2.138$	$L = 1.554$ $\lambda = 0.1$	$L = 1.880$ $\lambda = 0.25$	$L = 2.050$ $\lambda = 0.5$	$L = 2.107$ $\lambda = 0.75$	$H = 2.793$ $K = 0.4$	$H = 2.405$ $K = 0.5$	$L = 2.118$ $w = 2$	$L = 2.115$ $w = 3$	$L = 2.093$ $w = 4$	$L = 2.089$ $w = 5$	
0	MRL <sub>0</sub>	21	21	21	21	21	22	22	19	18	15	10
	ARL <sub>0</sub>	29.985	29.998	30.148	30.012	30.479	29.989	30.061	30.025	30.317	30.253	29.972
	SDRL <sub>0</sub>	30.075	27.232	28.370	28.618	29.290	27.055	27.434	32.420	35.677	40.820	44.793
0.0201	MRL <sub>1</sub>	18	15	15	16	16	16	16	16	13	10	7
	ARL <sub>1</sub>	26.074	19.672	20.609	22.320	23.595	21.579	22.310	23.735	22.598	20.906	19.370
	SDRL <sub>1</sub>	25.443	16.399	18.636	21.316	23.027	18.708	20.136	25.246	26.742	27.544	28.059
0.0705	MRL <sub>1</sub>	7	5	5	5	5	5	5	4	3	2	1
	ARL <sub>1</sub>	9.673	5.905	5.796	6.385	7.586	6.329	6.368	6.589	5.323	4.405	3.871
	SDRL <sub>1</sub>	9.188	3.242	3.790	5.078	6.797	3.923	4.308	6.700	5.822	5.164	4.731
0.1208	MRL <sub>1</sub>	3	3	3	3	3	3	3	2	1	1	1
	ARL <sub>1</sub>	3.965	3.360	3.040	2.969	3.223	3.338	3.214	2.575	2.105	1.823	1.662
	SDRL <sub>1</sub>	3.403	1.346	1.472	1.831	2.396	1.519	1.600	2.258	1.874	1.615	1.446
0.1510	MRL <sub>1</sub>	2	3	2	2	2	2	2	1	1	1	1
	ARL <sub>1</sub>	2.581	2.696	2.370	2.200	2.249	2.607	2.473	1.768	1.508	1.364	1.286
	SDRL <sub>1</sub>	2.022	0.948	1.013	1.181	1.464	1.040	1.080	1.321	1.070	0.891	0.769
0.2416	MRL <sub>1</sub>	1	2	1	1	1	2	1	1	1	1	1
	ARL <sub>1</sub>	1.256	1.792	1.483	1.299	1.244	1.642	1.516	1.084	1.045	1.029	1.022
	SDRL <sub>1</sub>	0.566	0.542	0.556	0.508	0.500	0.577	0.570	0.327	0.232	0.181	0.156
0.2719	MRL <sub>1</sub>	1	2	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.131	1.610	1.320	1.173	1.132	1.459	1.344	1.036	1.018	1.011	1.008
	SDRL <sub>1</sub>	0.386	0.527	0.482	0.393	0.363	0.528	0.496	0.204	0.139	0.110	0.094
0.3121	MRL <sub>1</sub>	1	1	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.048	1.403	1.164	1.073	1.051	1.266	1.180	1.010	1.004	1.003	1.002
	SDRL <sub>1</sub>	0.224	0.496	0.372	0.263	0.224	0.446	0.387	0.100	0.067	0.052	0.045
0.3524	MRL <sub>1</sub>	1	1	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.014	1.225	1.070	1.025	1.016	1.131	1.079	1.003	1.001	1.001	1.000
	SDRL <sub>1</sub>	0.121	0.418	0.256	0.158	0.127	0.338	0.270	0.052	0.033	0.026	0.020

**Table 11.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $ARL_0 = 50$

$\delta$	$\bar{X}$ $L = 2.326$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
		$L = 1.828$ $\lambda = 0.1$	$L = 2.120$ $\lambda = 0.25$	$L = 2.259$ $\lambda = 0.5$	$L = 2.317$ $\lambda = 0.75$	$H = 3.348$ $K = 0.4$	$H = 2.845$ $K = 0.5$	$L = 2.318$ $w = 2$	$L = 2.309$ $w = 3$	$L = 2.279$ $w = 4$	$L = 2.283$ $w = 5$	
0	MRL <sub>0</sub>	35	35	35	34	34	35	35	33	31	26	21
	ARL <sub>0</sub>	49.808	49.919	50.028	50.025	50.125	49.992	49.967	50.042	49.942	49.917	50.018
	SDRL <sub>0</sub>	48.490	47.111	48.855	48.822	48.755	46.422	47.275	52.063	57.946	65.356	71.267
0.0201	MRL <sub>1</sub>	28	21	22	24	27	23	23	26	21	15	12
	ARL <sub>1</sub>	41.066	27.918	30.799	34.493	38.283	31.069	31.948	37.806	34.747	30.959	29.717
	SDRL <sub>1</sub>	39.851	23.883	28.526	33.682	37.040	28.104	29.563	38.278	39.840	39.637	41.279
0.0705	MRL <sub>1</sub>	10	6	6	6	8	6	6	6	4	2	2
	ARL <sub>1</sub>	13.620	7.094	6.981	8.081	10.402	7.520	7.514	8.939	6.884	5.454	4.734
	SDRL <sub>1</sub>	12.895	3.729	4.622	6.579	9.509	4.500	4.952	9.299	7.648	6.457	5.873
0.1208	MRL <sub>1</sub>	4	4	3	3	3	4	3	2	1	1	1
	ARL <sub>1</sub>	5.077	3.909	3.474	3.447	3.964	3.857	3.669	3.084	2.408	2.024	1.821
	SDRL <sub>1</sub>	4.545	1.508	1.678	2.175	3.063	1.661	1.732	2.823	2.246	1.902	1.692
0.1510	MRL <sub>1</sub>	2	3	2	2	2	3	3	1	1	1	1
	ARL <sub>1</sub>	3.146	3.099	2.676	2.492	2.643	2.990	2.806	1.998	1.642	1.447	1.351
	SDRL <sub>1</sub>	2.598	1.048	1.111	1.349	1.791	1.116	1.145	1.589	1.261	1.028	0.893
0.2416	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.357	2.038	1.652	1.403	1.338	1.896	1.725	1.114	1.059	1.037	1.028
	SDRL <sub>1</sub>	0.694	0.522	0.587	0.574	0.590	0.569	0.593	0.389	0.274	0.207	0.176
0.2719	MRL <sub>1</sub>	1	2	1	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.187	1.859	1.465	1.247	1.189	1.696	1.525	1.050	1.024	1.014	1.011
	SDRL <sub>1</sub>	0.471	0.487	0.533	0.458	0.433	0.538	0.544	0.244	0.162	0.124	0.106
0.3121	MRL <sub>1</sub>	1	2	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.071	1.657	1.270	1.114	1.079	1.477	1.320	1.014	1.006	1.004	1.003
	SDRL <sub>1</sub>	0.277	0.497	0.449	0.322	0.278	0.513	0.474	0.123	0.080	0.060	0.051
0.3524	MRL <sub>1</sub>	1	1	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.024	1.452	1.133	1.043	1.027	1.285	1.165	1.004	1.002	1.001	1.001
	SDRL <sub>1</sub>	0.156	0.501	0.340	0.204	0.163	0.453	0.372	0.065	0.040	0.029	0.025



**Table 12.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $ARL_0 = 100$

$\delta$	$\bar{X}$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
	$L = 2.579$	$L = 2.168$	$L = 2.415$	$L = 2.532$	$L = 2.571$	$H = 4.188$	$H = 3.557$	$L = 2.569$	$L = 2.565$	$L = 2.535$	$L = 2.528$	
		$\lambda = 0.1$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$	$K = 0.4$	$K = 0.5$	$w = 2$	$w = 3$	$w = 4$	$w = 5$	
0	MRL <sub>0</sub>	69	67	69	71	70	69	68	69	62	56	53
	ARL <sub>0</sub>	100.011	100.067	100.333	100.029	100.029	100.387	100.387	100.084	99.685	100.285	99.985
	SDRL <sub>0</sub>	94.866	97.340	99.548	97.067	95.744	101.503	101.221	102.353	116.937	127.047	128.402
0.0201	MRL <sub>1</sub>	60	32	35	44	54	38	39	49	41.5	33	23
	ARL <sub>1</sub>	82.165	43.464	51.423	64.411	76.216	51.190	55.572	72.043	62.793	57.485	51.689
	SDRL <sub>1</sub>	81.497	37.346	50.615	63.180	76.273	45.523	53.463	74.347	70.345	71.418	70.546
0.0705	MRL <sub>1</sub>	16	8	7	8	11	8	8	9	6	4	2
	ARL <sub>1</sub>	23.030	8.774	8.885	11.358	15.865	9.316	9.384	13.448	9.884	7.541	6.310
	SDRL <sub>1</sub>	22.072	4.498	5.950	9.591	14.647	5.291	5.964	14.033	11.069	8.973	7.982
0.1208	MRL <sub>1</sub>	5	4	4	4	4	4	4	3	2	1	1
	ARL <sub>1</sub>	7.378	4.635	4.083	4.250	5.223	4.623	4.391	3.980	2.950	2.385	2.076
	SDRL <sub>1</sub>	6.848	1.708	1.955	2.763	4.213	1.841	1.928	3.820	2.915	2.385	2.072
0.1510	MRL <sub>1</sub>	3	3	3	3	3	3	3	1	1	1	1
	ARL <sub>1</sub>	4.291	3.639	3.071	2.935	3.270	3.568	3.327	2.400	1.872	1.597	1.451
	SDRL <sub>1</sub>	3.759	1.172	1.245	1.631	2.340	1.234	1.262	2.056	1.574	1.272	1.077
0.2416	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.543	2.319	1.872	1.564	1.482	2.224	2.040	1.164	1.083	1.051	1.036
	SDRL <sub>1</sub>	0.915	0.561	0.598	0.649	0.710	0.568	0.586	0.484	0.333	0.252	0.205
0.2719	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.291	2.109	1.665	1.366	1.283	2.015	1.834	1.076	1.035	1.020	1.014
	SDRL <sub>1</sub>	0.612	0.454	0.556	0.535	0.528	0.483	0.533	0.309	0.201	0.148	0.123
0.3121	MRL <sub>1</sub>	1	2	1	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.120	1.919	1.440	1.188	1.127	1.807	1.609	1.023	1.009	1.005	1.003
	SDRL <sub>1</sub>	0.367	0.403	0.511	0.400	0.350	0.464	0.516	0.158	0.098	0.073	0.059
0.3524	MRL <sub>1</sub>	1	2	1	1	1	2	1	1	1	1	1
	ARL <sub>1</sub>	1.042	1.753	1.253	1.082	1.048	1.613	1.398	1.007	1.003	1.001	1.001
	SDRL <sub>1</sub>	0.209	0.450	0.437	0.276	0.217	0.497	0.494	0.084	0.051	0.035	0.029

**Table 13.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $ARL_0 = 370$

$\delta$	$\bar{X}$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
		$L = 2.728$ $\lambda = 0.1$	$L = 2.899$ $\lambda = 0.25$	$L = 2.987$ $\lambda = 0.5$	$L = 2.977$ $\lambda = 0.75$	$H = 5.791$ $K = 0.4$	$H = 4.848$ $K = 0.5$	$L = 2.985$ $w = 2$	$L = 2.985$ $w = 3$	$L = 2.992$ $w = 4$	$L = 2.988$ $w = 5$	
0	MRL <sub>0</sub>	297.500	239.500	252	269.500	287.500	259	269	250	221.500	228.500	206
	ARL <sub>0</sub>	370.041	369.796	369.759	369.244	370.041	369.789	368.443	371.434	370.052	370.315	370.112
	SDRL <sub>0</sub>	310.944	368.139	358.695	339.327	317.665	367.869	361.766	359.217	416.009	428.953	453.432
0.0201	MRL <sub>1</sub>	227	73.500	100	156	163	87	94	163	136	126	89.500
	ARL <sub>1</sub>	296.721	97.631	137.119	208.135	224.139	113.184	128.183	232.481	212.113	190.658	173.318
	SDRL <sub>1</sub>	278.503	87.086	135.675	192.965	216.569	101.165	120.005	236.754	219.004	223.785	232.386
0.0705	MRL <sub>1</sub>	44	11	11	16	24	11	11	20	12	9	6
	ARL <sub>1</sub>	62.886	12.189	13.715	22.236	34.830	12.771	12.918	29.308	19.588	14.976	11.838
	SDRL <sub>1</sub>	60.794	6.195	9.922	19.940	33.010	6.572	7.695	30.362	21.695	17.853	15.025
0.1208	MRL <sub>1</sub>	11	6	5	5	6	6	5	5	2	2	1
	ARL <sub>1</sub>	16.023	5.987	5.290	6.265	8.844	6.114	5.713	6.702	4.376	3.406	2.817
	SDRL <sub>1</sub>	15.613	2.100	2.597	4.392	7.854	2.172	2.272	6.753	4.629	3.699	3.113
0.1510	MRL <sub>1</sub>	6	4	4	3	4	4	4	3	1	1	1
	ARL <sub>1</sub>	8.127	4.598	3.835	3.941	4.895	4.672	4.283	3.499	2.444	1.994	1.731
	SDRL <sub>1</sub>	7.643	1.390	1.568	2.300	3.796	1.432	1.452	3.293	2.295	1.849	1.537
0.2416	MRL <sub>1</sub>	2	3	2	2	2	3	2	1	1	1	1
	ARL <sub>1</sub>	2.088	2.818	2.219	1.893	1.808	2.828	2.548	1.299	1.140	1.086	1.060
	SDRL <sub>1</sub>	1.504	0.672	0.634	0.770	0.957	0.685	0.654	0.697	0.457	0.344	0.278
0.2719	MRL <sub>1</sub>	1	2	2	2	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.597	2.519	1.980	1.627	1.497	2.521	2.279	1.142	1.062	1.036	1.024
	SDRL <sub>1</sub>	0.973	0.578	0.535	0.639	0.702	0.584	0.532	0.441	0.280	0.206	0.163
0.3121	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.262	2.223	1.746	1.372	1.248	2.224	2.038	1.048	1.018	1.010	1.006
	SDRL <sub>1</sub>	0.572	0.444	0.504	0.517	0.483	0.447	0.409	0.236	0.140	0.100	0.081
0.3524	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.106	2.046	1.529	1.195	1.111	2.042	1.872	1.014	1.005	1.003	1.002
	SDRL <sub>1</sub>	0.341	0.312	0.510	0.402	0.324	0.319	0.400	0.123	0.073	0.051	0.040

**Table 14.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $MRL_0 = 30$

$\delta$	$\bar{X}$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
	$L = 2.264$	$L = 1.734$ $\lambda = 0.1$	$L = 2.050$ $\lambda = 0.25$	$L = 2.205$ $\lambda = 0.5$	$L = 2.258$ $\lambda = 0.75$	$H = 3.169$ $K = 0.4$	$H = 2.689$ $K = 0.5$	$L = 2.279$ $w = 2$	$L = 2.295$ $w = 3$	$L = 2.336$ $w = 4$	$L = 2.371$ $w = 5$	
0	MRL <sub>0</sub>	30	30	30	30	30	30	30	30	30	30	
	ARL <sub>0</sub>	42.556	41.941	43.041	43.528	43.152	42.710	41.887	45.283	48.348	58.266	63.563
	SDRL <sub>0</sub>	41.741	38.901	42.975	42.031	42.472	39.456	38.724	46.672	55.143	74.756	86.487
0.0201	MRL <sub>1</sub>	24	19	19	22	24	20	20	23	21	18	14
	ARL <sub>1</sub>	34.878	24.752	27.189	30.673	33.499	27.764	28.440	34.458	33.535	35.101	35.998
	SDRL <sub>1</sub>	33.699	20.950	25.027	29.115	32.097	24.528	26.032	35.019	38.514	45.191	49.801
0.0705	MRL <sub>1</sub>	9	6	5	6	7	6	6	6	4	2	2
	ARL <sub>1</sub>	12.124	6.679	6.606	7.590	9.505	7.138	7.103	8.432	6.762	5.839	5.225
	SDRL <sub>1</sub>	11.497	3.571	4.355	6.163	8.661	4.334	4.731	8.717	7.510	6.948	6.524
0.1208	MRL <sub>1</sub>	3	3	3	3	3	3	3	2	1	1	1
	ARL <sub>1</sub>	4.670	3.715	3.342	3.316	3.736	3.689	3.512	2.965	2.384	2.096	1.901
	SDRL <sub>1</sub>	4.105	1.459	1.620	2.081	2.845	1.623	1.694	2.695	2.215	2.002	1.808
0.1510	MRL <sub>1</sub>	2	3	2	2	2	3	2	1	1	1	1
	ARL <sub>1</sub>	2.938	2.958	2.585	2.410	2.518	2.867	2.687	1.948	1.631	1.477	1.385
	SDRL <sub>1</sub>	2.391	1.008	1.083	1.302	1.683	1.092	1.124	1.532	1.246	1.079	0.956
0.2416	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.319	1.958	1.602	1.374	1.310	1.816	1.648	1.108	1.058	1.039	1.030
	SDRL <sub>1</sub>	0.647	0.526	0.580	0.557	0.563	0.574	0.591	0.377	0.271	0.215	0.185
0.2719	MRL <sub>1</sub>	1	2	1	1	1	2	1	1	1	1	1
	ARL <sub>1</sub>	1.167	1.777	1.421	1.225	1.171	1.621	1.460	1.047	1.023	1.015	1.012
	SDRL <sub>1</sub>	0.443	0.505	0.521	0.441	0.413	0.541	0.533	0.236	0.161	0.129	0.112
0.3121	MRL <sub>1</sub>	1	2	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.063	1.571	1.236	1.102	1.069	1.406	1.265	1.013	1.006	1.004	1.003
	SDRL <sub>1</sub>	0.258	0.510	0.429	0.307	0.261	0.500	0.447	0.119	0.079	0.062	0.053
0.3524	MRL <sub>1</sub>	1	1	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.020	1.368	1.112	1.038	1.024	1.226	1.130	1.004	1.002	1.001	1.001
	SDRL <sub>1</sub>	0.145	0.484	0.316	0.191	0.153	0.420	0.338	0.062	0.039	0.030	0.027

**Table 15.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $MRL_0 = 50$

$\delta$	$\bar{X}$ $L = 2.447$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
		$L = 2.001$ $\lambda = 0.1$	$L = 2.281$ $\lambda = 0.25$	$L = 2.409$ $\lambda = 0.5$	$L = 2.434$ $\lambda = 0.75$	$H = 3.729$ $K = 0.4$	$H = 3.170$ $K = 0.5$	$L = 2.441$ $w = 2$	$L = 2.471$ $w = 3$	$L = 2.493$ $w = 4$	$L = 2.503$ $w = 5$	
0	MRL <sub>0</sub>	50	50	50	50	50	50	50	50	50	50	
	ARL <sub>0</sub>	68.777	71.778	71.315	72.716	69.479	70.164	69.145	69.530	77.029	89.672	94.237
	SDRL <sub>0</sub>	67.334	68.574	70.125	72.191	65.876	68.147	67.965	73.331	89.764	115.303	121.689
0.0201	MRL <sub>1</sub>	39	26	28	33	36	29	30	36	32	29	22
	ARL <sub>1</sub>	56.687	35.031	40.293	47.369	51.491	39.733	41.394	51.227	50.209	51.850	49.293
	SDRL <sub>1</sub>	55.142	29.988	37.894	47.067	50.252	36.384	38.299	52.273	57.479	64.759	66.875
0.0705	MRL <sub>1</sub>	12	7	7	7	9	7	7	7	5	4	2
	ARL <sub>1</sub>	17.217	7.923	7.957	9.714	12.542	8.325	8.374	10.858	8.611	7.124	6.122
	SDRL <sub>1</sub>	16.403	4.116	5.310	8.030	11.481	4.849	5.442	11.312	9.633	8.521	7.742
0.1208	MRL <sub>1</sub>	4	4	3	3	3	4	4	2	1	1	1
	ARL <sub>1</sub>	6.039	4.268	3.798	3.855	4.483	4.200	3.997	3.478	2.728	2.319	2.047
	SDRL <sub>1</sub>	5.560	1.605	1.816	2.470	3.527	1.735	1.818	3.261	2.642	2.297	2.028
0.1510	MRL <sub>1</sub>	3	3	3	2	2	3	3	1	1	1	1
	ARL <sub>1</sub>	3.631	3.378	2.884	2.725	2.907	3.250	3.042	2.175	1.780	1.570	1.440
	SDRL <sub>1</sub>	3.091	1.107	1.182	1.496	2.011	1.167	1.198	1.801	1.447	1.229	1.055
0.2416	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.436	2.181	1.773	1.488	1.399	2.049	1.876	1.136	1.073	1.048	1.035
	SDRL <sub>1</sub>	0.790	0.529	0.595	0.616	0.644	0.560	0.590	0.432	0.310	0.244	0.202
0.2719	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.231	1.994	1.575	1.308	1.229	1.852	1.669	1.062	1.031	1.019	1.013
	SDRL <sub>1</sub>	0.533	0.458	0.551	0.501	0.476	0.516	0.550	0.274	0.187	0.144	0.121
0.3121	MRL <sub>1</sub>	1	2	1	1	1	2	1	1	1	1	1
	ARL <sub>1</sub>	1.092	1.801	1.360	1.152	1.099	1.636	1.447	1.018	1.008	1.005	1.003
	SDRL <sub>1</sub>	0.318	0.453	0.489	0.366	0.311	0.508	0.511	0.139	0.091	0.070	0.058
0.3524	MRL <sub>1</sub>	1	2	1	1	1	1	1	1	1	1	1
	ARL <sub>1</sub>	1.031	1.613	1.193	1.062	1.035	1.426	1.260	1.005	1.002	1.001	1.001
	SDRL <sub>1</sub>	0.180	0.494	0.396	0.242	0.187	0.498	0.440	0.073	0.045	0.035	0.029

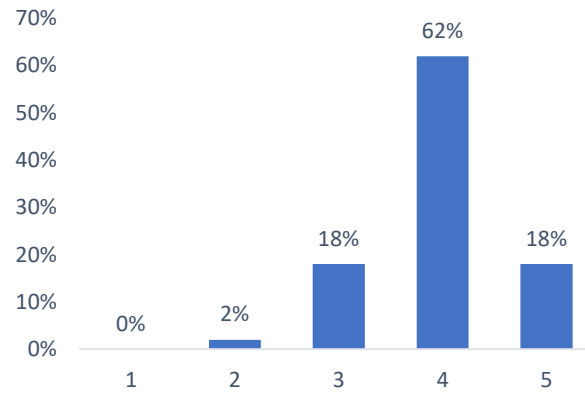
**Table 16.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $MRL_0 = 100$

$\delta$	$\bar{X}$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
	$L = 2.691$	$L = 2.331$	$L = 2.542$	$L = 2.679$	$L = 2.683$	$H = 4.635$	$H = 3.908$	$L = 2.669$	$L = 2.717$	$L = 2.739$	$L = 2.735$	
		$\lambda = 0.1$	$\lambda = 0.25$	$\lambda = 0.5$	$\lambda = 0.75$	$K = 0.4$	$K = 0.5$	$w = 2$	$w = 3$	$w = 4$	$w = 5$	
0	MRL <sub>0</sub>	100	100	100	100	100	100	101	100	100	100	
	ARL <sub>0</sub>	138.189	141.797	137.317	149.230	138.189	143.453	145.327	138.765	153.585	177.278	176.652
	SDRL <sub>0</sub>	129.534	133.933	130.883	149.438	132.448	142.356	143.991	137.568	175.088	219.220	225.081
0.0201	MRL <sub>1</sub>	75	40	45	63	70	48	50	66	61.500	57	44
	ARL <sub>1</sub>	111.976	54.212	65.338	90.222	101.518	65.087	72.291	92.761	93.195	97.529	85.244
	SDRL <sub>1</sub>	112.522	47.796	66.082	90.327	102.999	58.776	69.906	92.037	104.739	125.225	113.321
0.0705	MRL <sub>1</sub>	21	9	8	10	14	9	9	11	8	6	3
	ARL <sub>1</sub>	29.524	9.708	9.915	13.846	19.230	10.314	10.336	15.938	12.469	10.184	8.233
	SDRL <sub>1</sub>	28.812	4.961	6.725	11.859	17.797	5.745	6.425	16.852	13.878	12.220	10.522
0.1208	MRL <sub>1</sub>	6	5	4	4	5	5	4	3	2	1	1
	ARL <sub>1</sub>	8.879	5.018	4.367	4.786	6.002	5.040	4.747	4.457	3.366	2.769	2.356
	SDRL <sub>1</sub>	8.371	1.823	2.106	3.171	4.894	1.926	2.021	4.355	3.421	2.873	2.469
0.1510	MRL <sub>1</sub>	4	4	3	3	3	4	3	2	1	1	1
	ARL <sub>1</sub>	5.002	3.905	3.257	3.221	3.627	3.873	3.589	2.608	2.044	1.748	1.561
	SDRL <sub>1</sub>	4.484	1.232	1.322	1.814	2.662	1.293	1.317	2.292	1.793	1.491	1.265
0.2416	MRL <sub>1</sub>	1	2	2	2	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.652	2.458	1.963	1.662	1.560	2.386	2.183	1.190	1.100	1.065	1.045
	SDRL <sub>1</sub>	1.035	0.601	0.604	0.688	0.772	0.600	0.589	0.526	0.374	0.290	0.234
0.2719	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.354	2.223	1.750	1.443	1.333	2.155	1.967	1.088	1.043	1.026	1.018
	SDRL <sub>1</sub>	0.691	0.487	0.554	0.572	0.574	0.486	0.508	0.335	0.227	0.172	0.140
0.3121	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.147	2.013	1.520	1.239	1.154	1.946	1.748	1.028	1.012	1.007	1.005
	SDRL <sub>1</sub>	0.411	0.380	0.521	0.442	0.384	0.413	0.491	0.175	0.112	0.084	0.068
0.3524	MRL <sub>1</sub>	1	2	1	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.054	1.864	1.319	1.112	1.062	1.775	1.539	1.008	1.003	1.002	1.001
	SDRL <sub>1</sub>	0.238	0.388	0.470	0.318	0.245	0.444	0.507	0.092	0.058	0.042	0.033

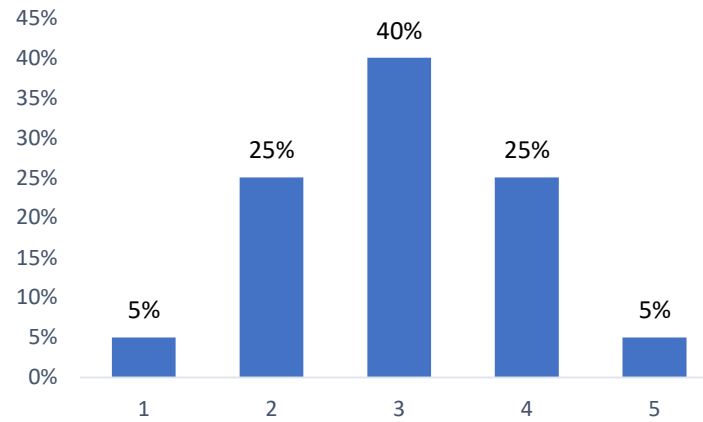
**Table 17.** ARL, SDRL and MRL values of  $\bar{X}$ , EWMA, CUSUM  $\bar{X}$  and MA charts for various  $\delta$  values when  $MRL_0 = 370$

$\delta$	$\bar{X}$ $L = 3.086$	EWMA				CUSUM $\bar{X}$		Moving Average (MA)				
		$L = 2.900$ $\lambda = 0.1$	$L = 3.057$ $\lambda = 0.25$	$L = 3.083$ $\lambda = 0.5$	$L = 3.068$ $\lambda = 0.75$	$H = 6.259$ $K = 0.4$	$H = 5.298$ $K = 0.5$	$L = 3.100$ $w = 2$	$L = 3.136$ $w = 3$	$L = 3.154$ $w = 4$	$L = 3.131$ $w = 5$	
0	MRL <sub>0</sub>	369.500	368.500	367	369	371	375	370	372	371	372	370
	ARL <sub>0</sub>	468.100	594.333	577.029	498.357	481.422	542.630	536.817	533.188	601.055	605.177	628.158
	SDRL <sub>0</sub>	407.502	579.828	588.635	484.531	431.333	530.751	490.484	502.305	661.158	719.177	681.176
0.0201	MRL <sub>1</sub>	261	91	142	187	216	98	118	230	226	188.500	134
	ARL <sub>1</sub>	356.817	127.854	201.089	260.849	291.088	136.975	170.472	334.338	330.811	304.588	247.227
	SDRL <sub>1</sub>	331.796	115.304	194.675	250.175	270.711	127.186	158.056	354.170	321.719	366.706	315.830
0.0705	MRL <sub>1</sub>	53	12	13	19	29	12	12	25	17	12	8
	ARL <sub>1</sub>	75.577	13.431	15.915	25.926	42.462	13.776	14.114	37.215	26.021	19.716	14.675
	SDRL <sub>1</sub>	71.976	6.854	12.099	23.566	41.881	6.927	8.213	38.236	28.806	23.425	18.789
0.1208	MRL <sub>1</sub>	13	6	5	5	7	6	6	5	3	2	1
	ARL <sub>1</sub>	18.571	6.437	5.784	6.865	10.077	6.566	6.170	7.839	5.194	3.930	3.139
	SDRL <sub>1</sub>	18.114	2.226	2.884	4.870	8.958	2.255	2.372	7.982	5.591	4.364	3.557
0.1510	MRL <sub>1</sub>	7	5	4	4	4	5	4	3	1	1	1
	ARL <sub>1</sub>	9.235	4.914	4.118	4.212	5.438	5.000	4.613	3.958	2.750	2.193	1.849
	SDRL <sub>1</sub>	8.7	1.471	1.682	2.51	4.292	1.488	1.511	3.784	2.688	2.115	1.719
0.2416	MRL <sub>1</sub>	2	3	2	2	2	3	3	1	1	1	1
	ARL <sub>1</sub>	2.229	2.990	2.332	1.970	1.901	3.019	2.725	1.352	1.169	1.103	1.069
	SDRL <sub>1</sub>	1.657	0.695	0.656	0.796	1.026	0.700	0.690	0.776	0.513	0.385	0.303
0.2719	MRL <sub>1</sub>	1	3	2	2	1	3	2	1	1	1	1
	ARL <sub>1</sub>	1.675	2.662	2.075	1.686	1.558	2.682	2.425	1.168	1.076	1.044	1.028
	SDRL <sub>1</sub>	1.058	0.604	0.538	0.657	0.745	0.608	0.571	0.489	0.317	0.231	0.180
0.3121	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.298	2.332	1.836	1.420	1.285	2.346	2.149	1.058	1.023	1.012	1.007
	SDRL <sub>1</sub>	0.619	0.493	0.485	0.536	0.517	0.499	0.428	0.262	0.158	0.113	0.088
0.3524	MRL <sub>1</sub>	1	2	2	1	1	2	2	1	1	1	1
	ARL <sub>1</sub>	1.123	2.115	1.631	1.227	1.130	2.124	1.978	1.018	1.007	1.003	1.002
	SDRL <sub>1</sub>	0.371	0.35	0.501	0.427	0.35	0.36	0.343	0.140	0.084	0.059	0.044

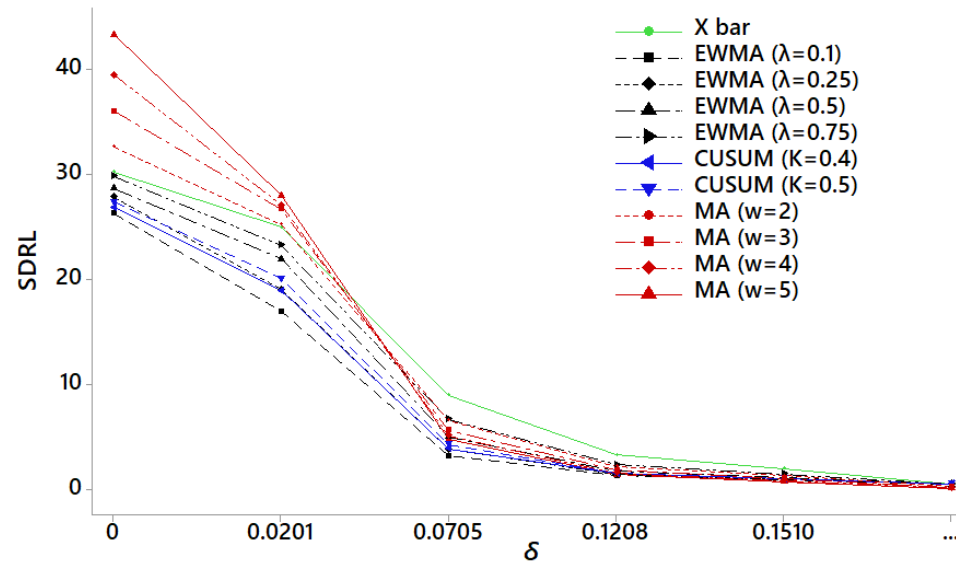
**Figure 3.** Distribution of student evaluations about the overall performance of a faculty member



**Figure 4.** Imaginary distribution of student evaluations about the overall performance of a faculty member



**Figure 5.** SDRL for various  $\delta$  values, for the real data distribution of student evaluations, in case  $ARL_0 = 30$  for all compared CCs



**Figure 6.** ARL for various  $\delta$  values, for the real data distribution of student evaluations, in case  $MRL_0 = 100$  for all compared CCs

